LARGE-EDDY SIMULATION OF A SHEAR-FREE MAGNETOHYDRODYNAMIC MIXING LAYER

O. DEBLIQUY, B. KNAEPEN AND D. CARATI
Univeristé Libre de Bruxelles, Statistical and Plasmas Physics,
Bld du Triomphe, Campus Plaine - CP 231,
B-1050 Brussels, Belgium

Abstract. We present LES results of the evolution of a decaying magnetohydrodynamic (MHD) mixing layer using dynamic eddy-viscosity subgrid scale models. The LES results are obtained using a spectral code with a 32^3 resolution and are compared to a direct numerical simulation (DNS) with 128^3 Fourier modes. The evolution of the kinetic and magnetic energies is presented and their profiles along the inhomogeneous direction is also discussed.

1. Introduction

MHD is recognized as a valid approximation in various problems of plasma physics such as nuclear fusion, astrophysics, geophysics, ... In many cases, highly turbulent processes are encountered and the magnetic Reynolds number R_m characterizing the magnetic turbulence can reach values ranging from 10^8 to 10^{12}. For such values, the use of DNS for investigating the MHD turbulence is inappropriate. In this context, developing the LES technique, which has been already widely used in fluid mechanics, appears to be an elegant solution for solving the incompressible MHD equations:

$$\partial_t u_i = -\partial_j(u_j u_i - b_j b_i) + \nu\nabla^2 u_i - \partial_i p \quad (1)$$

$$\partial_t b_i = -\partial_j(u_j b_i - b_j u_i) + \eta\nabla^2 b_i \quad (2)$$

where $b_i = B_i/\sqrt{\rho\mu_0}$ denotes the components of the reduced magnetic field, ρ is the constant density and p represents the sum of the hydrodynamic and magnetic pressure $b_l b_l/2$ usually evaluated by enforcing the incompressibility condition ($\partial_i u_i = 0$). The parameters ν and η are the kinematic

viscosity and the magnetic diffusivity, respectively. Although LES has already been adapted to MHD (Theobald et al., 1994; Agullo et al., 2001; Müller and Carati, 2001), its use has been limited to homogeneous turbulence. In this work, we explore the capabilities of this technique for inhomogeneous flows. The particular case treated here is the mixing layer. Our choice is motivated by the fact that interactions between regions of different turbulent activities are very commonly observed in many geophysical and astrophysical problems. Our choice is however also motivated by practical computational arguments. Indeed, the mixing layer can be computed with a spectral code (fully de-aliased) for which the modelling issues do not interfere too strongly with the numerics. The mixing-layer we have considered is the interface between two regions of almost homogeneous turbulence with different mean energy and different energy spectra. In our case, the direction of inhomogeneity will be oriented along the y-axis. The u_i and b_i fields are initialized to resemble a magnetohydrodynamic mixing layer by adapting the well-documented hydrodynamic case of Veeravalli and Warhaft (1989) (see also, Briggs et al., 1996).

2. LES equations

Within the framework of LES, a filter kernel is applied to the MHD equations in order to obtain a set of equations for the resolved quantities. Here, because our code is spectral, we adopt the sharp fourier cut-off for the filtering operator and the filter width is noted by $\overline{\Delta}$. The filtered MHD equations thus read:

$$\partial_t \overline{u}_i = -\partial_j(\overline{\overline{u}_j\overline{u}_i} - \overline{\overline{b}_j\overline{b}_i}) + \nu\nabla^2\overline{u}_i - \partial_i\overline{p} - \partial_j\overline{\tau}^u_{ji} \quad (3)$$

$$\partial_t \overline{b}_i = -\partial_j(\overline{\overline{u}_j\overline{b}_i} - \overline{\overline{b}_j\overline{u}_i}) + \eta\nabla^2\overline{b}_i - \partial_j\overline{\tau}^b_{ij} \quad (4)$$

In contrast with traditional notations, we have explicitly expressed that the non-linear term is filtered since our code is fully de-aliased. Also, we have explicitly written the filtering operator on the subgrid-scale stress tensors $\overline{\tau}^u_{ij} = (\overline{u_iu_j} - \overline{\overline{u}_i\overline{u}_j}) - (\overline{b_ib_j} - \overline{\overline{b}_i\overline{b}_j})$ and $\overline{\tau}^b_{ij} = (\overline{u_ib_j} - \overline{\overline{u}_i\overline{b}_j}) - (\overline{u_jb_i} - \overline{\overline{u}_j\overline{b}_i})$ for two reasons. First, this might avoid some confusion because the notation τ_{ij} usually refers in the Navier-Stokes case to the term $\overline{u_iu_j} - \overline{u}_i\overline{u}_j$. Second, since $\overline{\tau}_{ij}$ has to be computed on the LES grid, it is unavoidably a filtered quantity. Those terms account for the effects of the small scales on the large scales and cannot be computed directly from the resolved quantities. Therefore, in order to close the equation (3) and (4) , we need to model them.

The model proposed here is based on the eddy-viscosity assumption and is referred to as the Kolmogorov model (Agullo et al., 2001). One

uses therefore a MHD-generalization of the Smagorinsky model with Kolmogorov scaling for the eddy-diffusivities:

$$\overline{\tau}^u_{ij} \approx -2C_1\overline{\Delta}^{4/3}\overline{S}_{ij} \tag{5}$$

$$\overline{\tau}^b_{ij} \approx -2C_2\overline{\Delta}^{4/3}\overline{W}_{ij} \tag{6}$$

where $\overline{S}_{ij}$ is the symmetric part of the filtered velocity gradients and $\overline{W}_{ij}$ is the anti-symmetric part of the filtered magnetic field gradients. We have adopted the MHD-extension of the *dynamic procedure* (Germano et al., 1991; Lilly, 1992) for computing the parameters C_1 and C_2. To that end, we introduce a second filter referred to as the test-filter and whose action is noted by $\widehat{\ }$. The test filter is also a Fourier cut-off with $\widehat{\Delta} = 2\overline{\Delta}$. Because of the properties of the Fourier cut-off filters, the following relation $\widehat{\overline{\cdots}} \equiv \widehat{\cdots}$ can be used to simplify the notation. The application of the filter $\widehat{\cdots}$ to the MHD equations introduces two additional unknown stress tensors, $\widehat{T}^u_{ij}$ and $\widehat{T}^b_{ij}$. They will be referred to as the subtest-stress tensors and their definitions are similar to $\overline{\tau}^u_{ij}$ and $\overline{\tau}^b_{ij}$ for test-level quantities. They are assumed to be modeled as follows:

$$\widehat{T}^u_{ij} \approx -2C_1\widehat{\Delta}^{4/3}\widehat{S}_{ij} \tag{7}$$

$$\widehat{T}^b_{ij} \approx -2C_2\widehat{\Delta}^{4/3}\widehat{W}_{ij}\,. \tag{8}$$

The Germano identities obtained for the stresses $\overline{\tau}$ and $\widehat{T}$ thus read:

$$\widehat{T}^u_{ij} - \widehat{\overline{\tau}}^u_{ij} = \widehat{L}^u_{ij} \tag{9}$$

$$\widehat{T}^b_{ij} - \widehat{\overline{\tau}}^b_{ij} = \widehat{L}^b_{ij} \tag{10}$$

where $\widehat{L}^u_{ij} = (\widehat{\overline{u}_i\overline{u}_j} - \widehat{u}_i\widehat{u}_j) - (\widehat{\overline{b}_i\overline{b}_j} - \widehat{b}_i\widehat{b}_j)$ and $\widehat{L}^b_{ij} = (\widehat{\overline{u}_i\overline{b}_j} - \widehat{u}_i\widehat{b}_j) - (\widehat{\overline{b}_i\overline{u}_j} - \widehat{b}_i\widehat{u}_j)$. These expressions can be used to evaluate C_1 and C_2 if we assume that C_1 and C_2 are independent of the filter width. Here, C_1 and C_2 are also assumed to be function of the inhomogeneous direction, i.e. the y-direction, and are chosen to minimize the errors defined as

$$Q_1(y) = \langle(\widehat{T}^u_{ij} - \widehat{\overline{\tau}}^u_{ij} - \widehat{L}^u_{ij})^2\rangle_{xz} \tag{11}$$

$$Q_2(y) = \langle(\widehat{T}^b_{ij} - \widehat{\overline{\tau}}^b_{ij}\widehat{L}^b_{ij})^2\rangle_{xz}\,, \tag{12}$$

where $\langle\,\rangle_{xz}$ represents the average over the xz-plan.

3. Initial Conditions

The velocity and magnetic fields are initialized using the same procedure. We will thus discuss this procedure for a generic field denoted c. In the

following, both the three-dimensional $c(k_x, k_y, k_z)$ and the two-dimensional $\tilde{c}(k_x, y, k_z)$ Fourier transforms of this field are used. For isotropic turbulence, the averaged amplitude of $c(k_x, k_y, k_z)$ only depends on the norm of the wave vector $k = \sqrt{k_x^2+k_y^2+k_z^2}$: $\langle|c(k_x, k_y, k_z)|^2\rangle = A^2(k)$. Also, the averaged amplitude of $\tilde{c}(k_x, y, k_z)$ only depends on the norm of the wave vector perpendicular to the y-axis $k_\perp = \sqrt{k_x^2+k_z^2}$: $\langle|c(k_x, y, k_z)|^2\rangle = B^2(k_\perp)$. The relation between these two amplitudes is readily derived from the Parseval theorem:

$$B^2(k_\perp) = \int_{-\infty}^{+\infty} dk_y A^2(\sqrt{k_\perp^2 + k_y^2}), \tag{13}$$

We also know that for isotropic turbulence, the field amplitude $A^2(k)$ is related to the energy spectrum $E(k) = 2\pi k^2 A^2(k)$. Hence, if the statistical properties of the field slowly vary along the axis y, we can assume, in a first approximation, that:

$$B^2(k_\perp, y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dk_y \frac{E(\sqrt{k_\perp^2 + k_y^2}, y)}{k_\perp^2 + k_y^2} \tag{14}$$

In our case, the energy spectrum will be given by

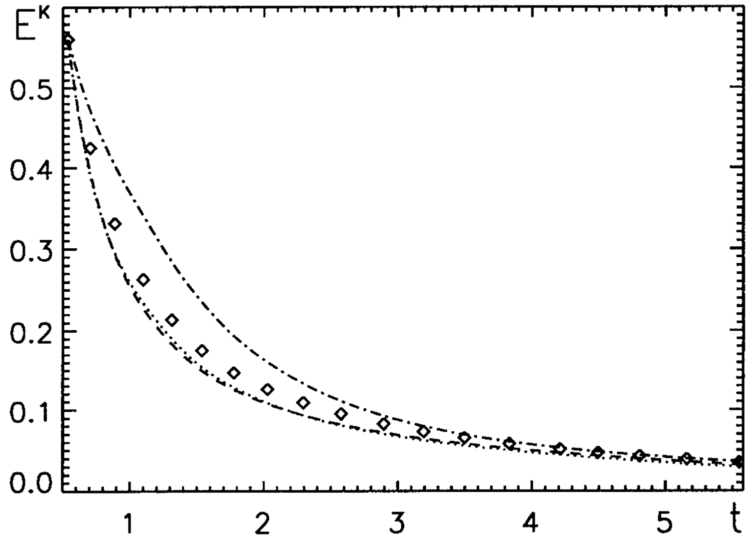
$$E(k, y) = A \frac{k^4 e^{-k^2/\alpha^2}}{(q^4 + k^4)^{17/12}}. \tag{15}$$

The y-dependence is controlled through the parameters: $A = A(y)$, $\alpha = \alpha(y)$ and $q = q(y)$. The values of these parameters have been chosen so that the initial conditions mimick the experimental fields produced by Veeravalli and Warhaft (1989) using the 3:1 grid. They are constant in the two regions corresponding to the quasi-homogeneous layers and vary continuously in the mixing layer in order to connect smoothly the two quasi-homogeneous layers. They are chosen so that the ratio of turbulence intensities is about 6 while the ratio of typical lengths (corresponding to the spectral energy peak) is about 3.

The velocity and magnetic field are initially uncorrelated. However, turbulent phases are build using hundred time steps for which both u_i and b_i are advanced in time and then rescaled to the desired amplitudes $B^2(k_\perp, y)$.

4. Results

In order to assess the LES results, we have performed a 128^3 DNS in which the u_i and b_i fields are initialized with the procedure described in the previous section. The same spectra and the same set of parameters as in (15) have been chosen for the velocity and magnetic fields. The kinematic viscosity and the magnetic diffusivity are identical ($\nu = \eta = 4.10^{-3}$) so that



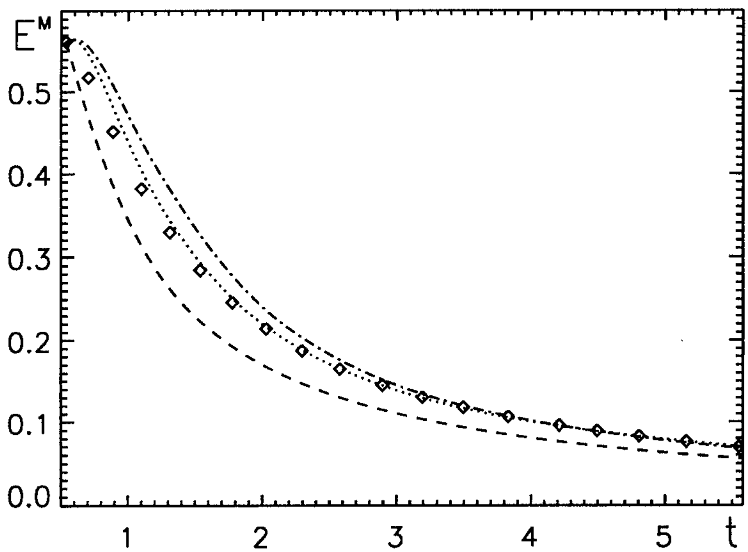


Figure 1. Evolution of the kinetic (top) and magnetic (bottom) energies as a function of time using the K-K model (dashed line), the K-NO model (dotted line) and the NO-NO model (dot-dashed line). The DNS filtered to 32^3 modes is represented by the symbol $\diamond$.

the Prandtl number=1. We present results for three types of LES. The first one, hereafter referred to as the K-K model, uses the Kolmogorov scaling (5) in both the equations for $\overline{u}_i$ and $\overline{b}_i$. It was however realized that this model is too strongly dissipative as long as the magnetic energy is concerned. This has motivated the used of the Kolmogorov model for the

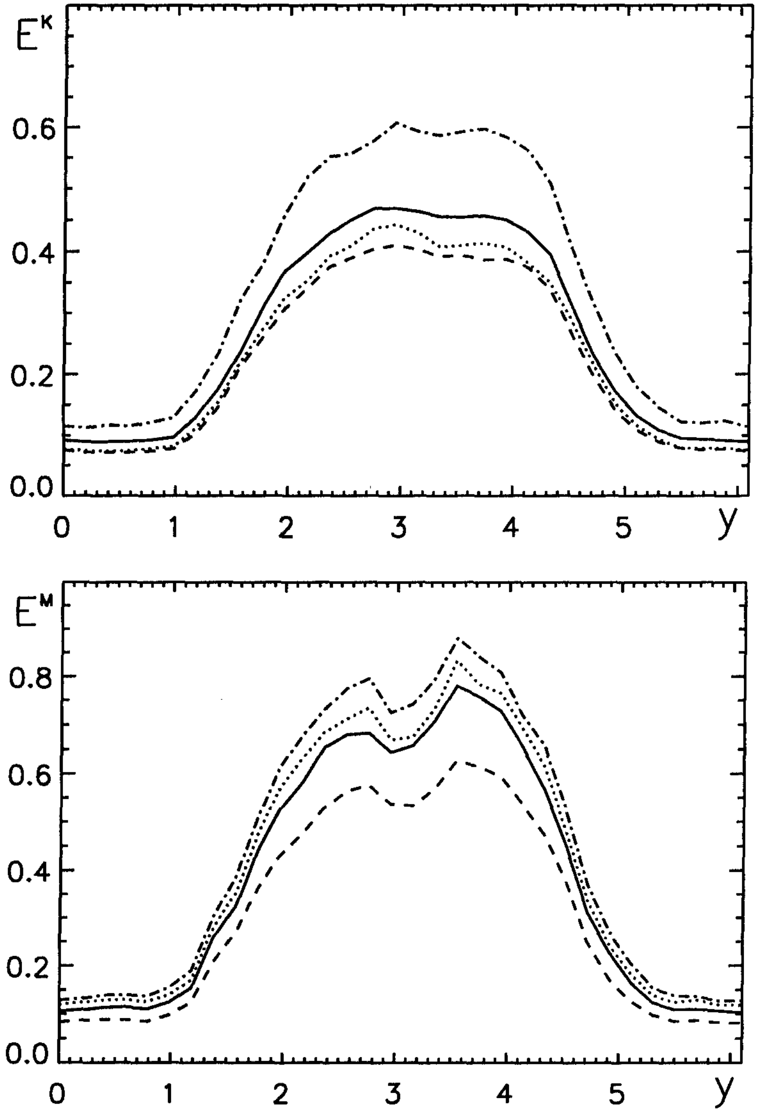


Figure 2. Profile of the kinetic (top) and magnetic (bottom) energies along the inhomogeneous y-direction at $t = 1.1$. The DNS filtered to 32^3 modes is represented by the solid line.

velocity equation only, while neglecting the effect of the subgrid-scale in the equation for $\bar{b}_i$. This model is referred to as the K-NO model. Finally, in order to emphasize the importance of the model, results obtained without any model (referred to as the NO-NO model) are also presented. Figure 1 shows the decay of the volume averaged kinetic and magnetic energies as a function of time.

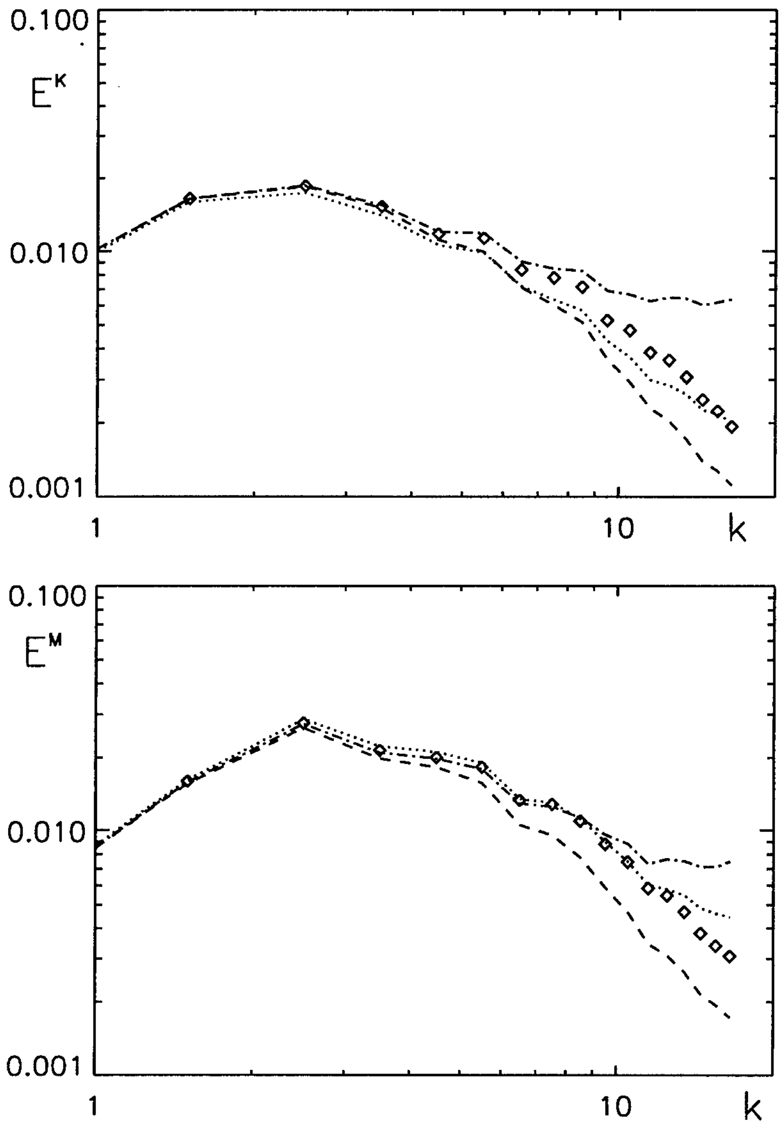


Figure 3. Kinetic (top) and magnetic (bottom) energy spectra at $t = 1.1$. Symbols are the same as in Fig. 1.

We can see that the K-K model predictions agree reasonably well with the filtered DNS as long as the kinetic energy is concerned. However, this model appears to dissipate too much magnetic energy. The K-NO model has a much reasonable behavior for the magnetic energy and it even improves slightly the prediction of the kinetic energy.

Figure 2 shows the profile of the energies along the anisotropic direction

at time $t = 1.1$ for which about 50% of the initial energy has been dissipated. We observe the same trends as in Figure 1, i.e. the kinetic energy profile predicted by the K-K model is quite close from the filtered DNS results, while the magnetic energy profile is significantly below the DNS filtered results. Again, using the K-NO model significantly improves the agreement with DNS data for both the energy profiles.

We also present the energy spectra at the same time which are quantities rather sensitive to the modeling (Figure 3). Indeed, for the NO-NO model, the expected piling up of the energy in the high wave vector modes is observed for both the kinetic and magnetic energies. Here also the best performances are obtained when using the K-NO model.

5. conclusion

We have performed a preliminary study of a magnetohydrodynamic mixing-layer LES. We have proposed the use of the Kolmogorov model for which the parameters were computed by the MHD-extended dynamic procedure with an explicit dependence of the parameters on the inhomogeneity direction.

The results clearly show that the K-NO model outperforms significantly the other models. This seems to indicate that the modelling of the subgrid-scales in much more important for the velocity than for the magnetic field. Our preliminary results seem also to demonstrate that the effects of the subgrid-scales in the magnetic field equation cannot be appropriately modelled in terms of a simple eddy magnetic diffusivity.

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